

Transforming 2 X 2 Games

Game Column payoffs **Graph**

Row payoffs 1,4

Nash Equilibrium

Prisoner's Dilemma

Maximin

Pareto-deficient

Incentives & externalities

Pareto-optimal outcomes in bold

Additional nonmyopic equilibrium # moving power

Threat # compellent # deterrent

Mixed Strategy: (down, left) 3/4, 1/2

Payoffs 2.5, 2.5 iff ratio or real values

Payoff swaps change one ordinal game into another

Adjacent games are neighbors by payoff swaps

1↔2 swaps form tiles of 4 games

2↔3 swaps join tiles into 4 layers

3↔4 swaps link four layers

Layers differ by alignment of 4s

Each layer is a torus, table is a torus

Layers scrolled to center Prisoner's Dilemma

Tile



4 Layers 1



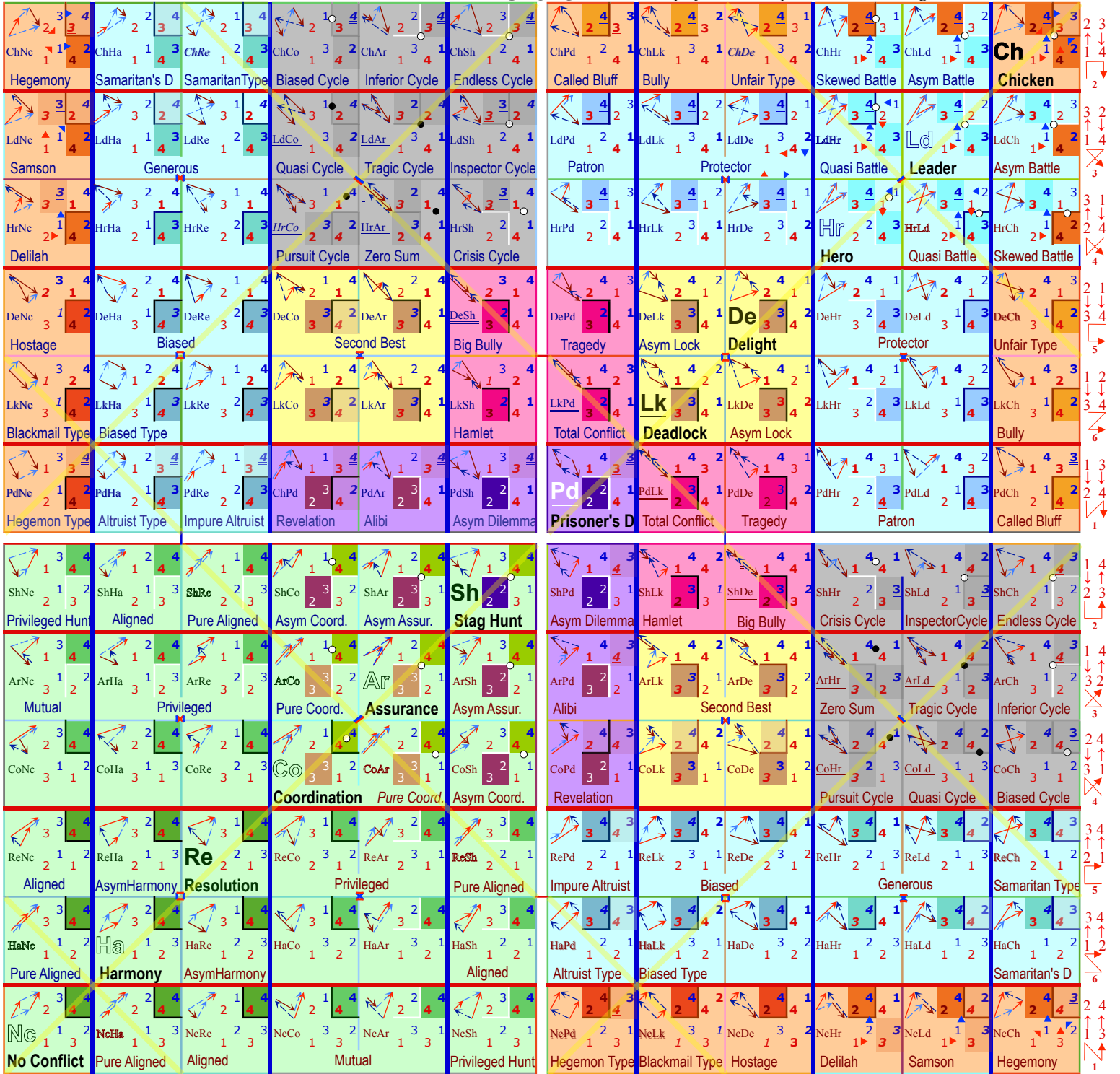
Families	Harmonious	
1. Win-win 4,4	Stag Hunt	
2. Biased 4,3	Battle	
Self-serving		Samaritan
3. Second Best 3,3		
4. Unfair 4,2	Winner	Loser
5. Dilemma	Alibi 3,2	
Prisoner's D. 2,2	Tragic 3,2	
6. Cyclic (2.5,2.5)		

4

Symmetric Pure Cooperation Pure Conflict Fixed Sum Jekyll-Hyde Type

Chart

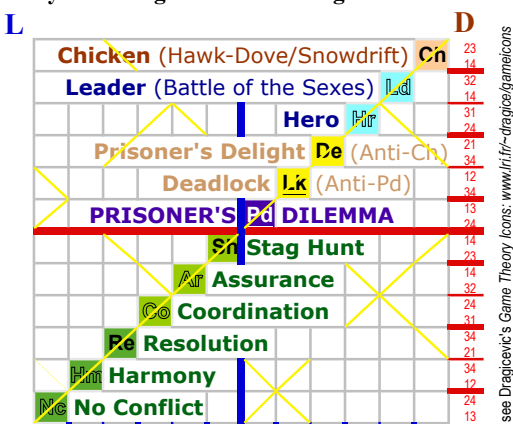
A map of relationships between the 2x2 games



To find a game: Make ordinal 4>3>2>1. Put column with Row's 4 right, row with Column's 4 up. Find layer by alignment of 4s; then intersection of Row & Column payoffs.

Structures in the Topology of 2x2 Games

a. Symmetric games form a diagonal axis



see Dragilevic's Game Theory icons: www.fir-drageice.com/icons

Payoffs: 23, 14, 14, 14, 24, 34, 31, 21, 21, 12, 12, 13, 24, 14, 14, 24, 34, 31, 21, 21, 34, 43, 42, 41, 41, 42, 43, 43, 12, 13, 12

W R

Symmetric Quasi-symmetric Sub-symmetric

d. High swaps (3↔4) link layers

connecting equivalently-located tiles

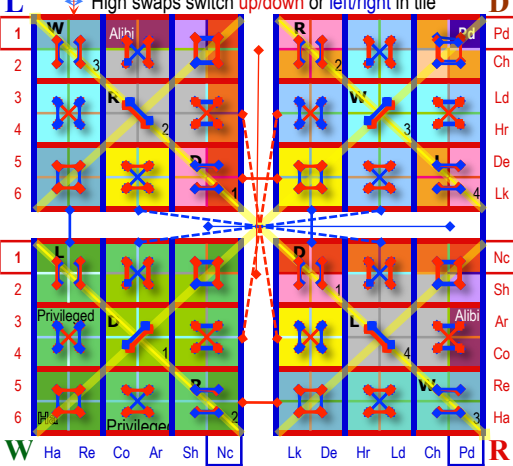
6 Hotspots double-link pairs of tiles

13 Win-win-Discord (Coord.-Battle)

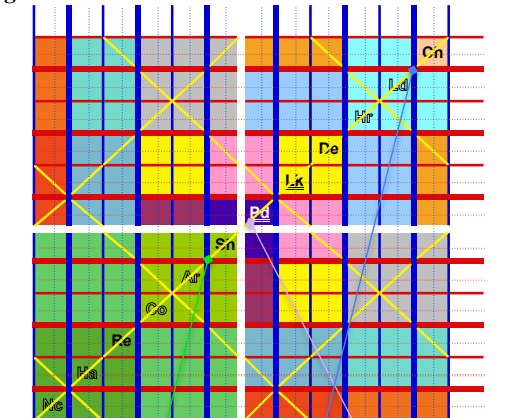
6 Pipes link stacks of 4 tiles on 4 layers

Pd Pipe (Pd scrolled to NE corner to unify tiles)

High swaps switch up/down or left/right in tile



g. Games with ties form borderlines



Games with ties (non-strict) lie between strict ordinal games, at grid intersections (graph nodes/vertices)

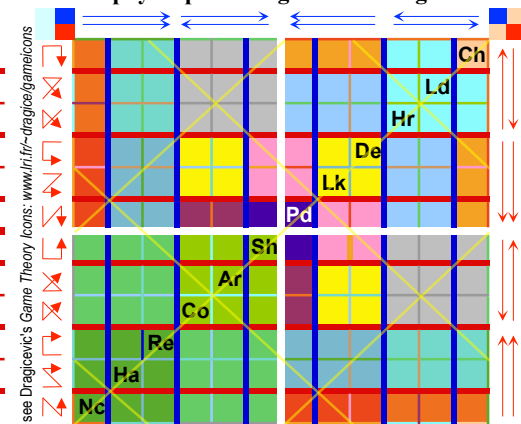
Strict Half-swaps: 2=1, 2=3, 3=4

see Robinson, Goforth and Cargill 2007

The topology also maps all normalized 2x2 games

mSh Mid Stag Hunt, mVo Mid Volunteer's D, Ax Axelrod's Pd

b. Twelve payoff patterns generate 144 games

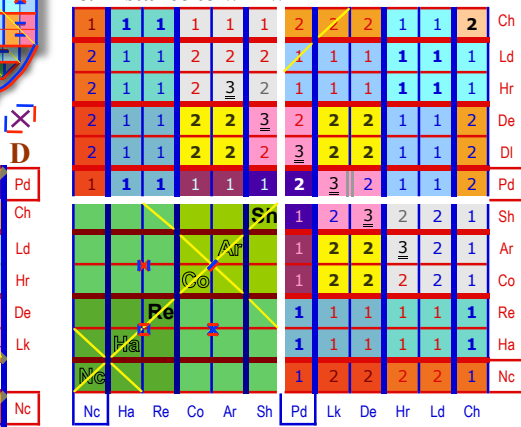


Reflections around axis switch row & column positions

4 games per tile, 36 games and 9 tiles per layer

66 asymmetric pairs: 66 + 12 = 78 "unique" games

e. Distance to win-win



Swaps to reach win-win 1= single 3↔4 swap

2 or 3 step paths may include 2↔3 and 1↔2 swaps

Bold = paths for both. Pareto-efficient paths, each step results in same or better-ranked outcome

Zero-sum games are hardest to remedy

h. Rapoport & Guyer Taxonomy

L	Nc	Ha	Re	Co	Ar	Sh	Pd	Lk	De	Hr	Ld	Ch	D
2	55	50	49	70	78	72	39	35	36	65	67	66	Ch
3	56	52	51	74	76	71	37	31	32	64	68	67	Ld
4	44	41	40	73	75	77	38	33	34	69	64	65	Hr
5	18	16	15	53	42	45	10	8	7	34	32	35	De
6	17	14	13	54	43	46	11	9	8	33	31	35	Lk
1	21	19	20	57	47	48	12	10	10	38	37	39	Pd
2	26	22	23	58	62	61	48	46	45	77	71	72	Sh
3	27	24	25	59	63	62	47	43	42	75	76	78	Ar
4	30	28	29	60	59	58	57	54	53	73	74	70	Co
5	2	4	5	29	25	23	20	13	15	40	51	49	Re
6	1	3	4	28	24	22	19	14	16	41	52	50	Ha
1	3	2	3	30	27	26	21	17	18	44	56	55	Nc

W R

No Conflict N EP D_{2,0} D_{1,0} D_{0,0} D_{2,c} D_{1,c} D_{0,c} E=Natural outcome is Equilibrium

Mixed Motive M EP D_{2,0} D_{1,0} D_{1,t} D_{1,f} D_{0,f} D_{1,tf} D_{1,ctf} 2,1,0 Dominant strategies

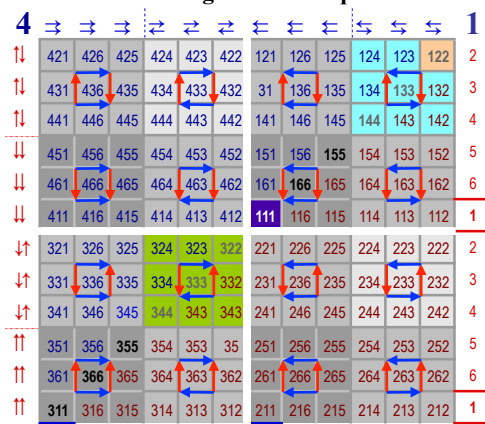
M e no D₀ 2 equilibria e_{2,0} e_{2,c} competitive pressure

M e dominance D₀ No equilibria e_{0,0} e_{0,c} threat-vulnerable

Complete ZE D_{2,0} D_{1,0} e eD_{0,c} force-vulnerable

Opposition see Rapoport et al. 1976 2x2 Games; Robinson & Goforth 2003

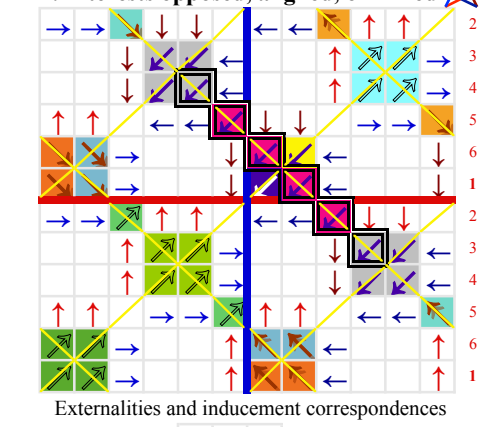
c. Dominant strategies lead to equilibria



Dominant strategies D₁ Column D₀ None D₂ Both D₁ Row

Nash Equilibria 1 0/2 (for ordinal) 1 1 payoffs)

f. Interests opposed, aligned, or mixed



Externalities and inducement correspondences

Jekyll-Hyde Type ++ ↑ ↗ Pure Cooperation

Pure Conflict -- ↓ ↘ Jekyll-Hyde Type

Fixed Rank-sum ± ± ++

(Zero Sum) see Schelling 1963 The Strategy of Conflict Greenberg 1990; Robinson & Goforth 2005

i. Brams Typology and Game Numbers

4	Nc	Ha	Re	Co	Ar	Sh	Pd	Lk	De	Hr	Ld	Ch	1
Ch	50	37	36	46	31	29	22	18	19	52	53	57	2
Ld	56	39	38	43	45	47	20	14	15	51	54	53	3
Hr	49	13	12	42	44	30	21	16	17	55	51	52	4
De	6	4	3	40	23	25	10	8	7	17	15	19	5
Lk	5	2	1	41	24	26	11	9	8	16	14	18	6
Pd	35	33	34	48	27	28	22	11	10	21	20	22	1
Sh	Number of non-myopic equilibria (NMEs)						28	26	25	30	47	29	2
Ar	All Nash equilibria are NMEs except asymmetric						27	24	23	44	45	31	3
Co	pareto-deficient: 27,28,48						48	41	40	42	43	46	4
Re							34	1	3	12	38	36	5
Ha							33	2	4	13	39	37	6
Nc							35	5	6	49	56	50	1

3 1 6 5 4 3 2 1 6 5 4 3 2 2

Strongly cyclic

Not cyclic Moderately cyclic

No conflict Weakly cyclic

see Brams 1994 Theory of Moves; Difficult see Brams & Kilgour 2008